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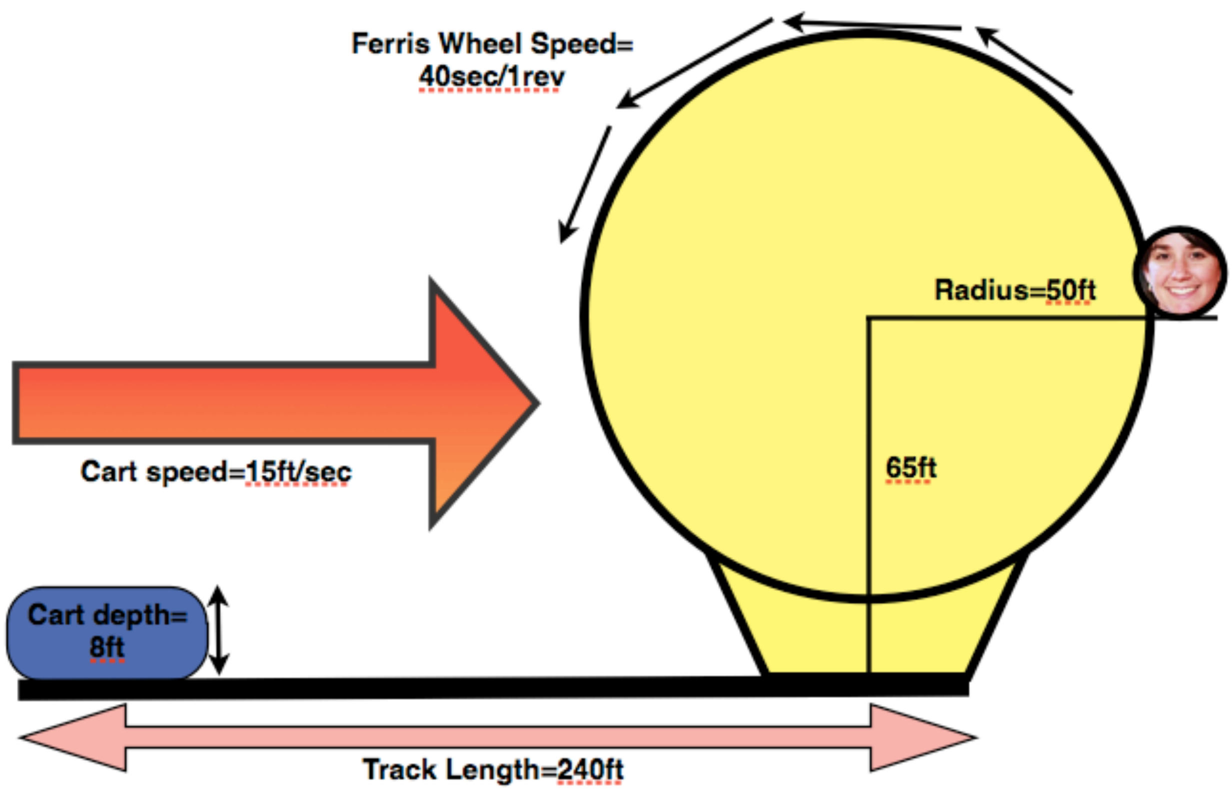
C BLOCK

Ferris Wheel Dive

As a special graduation present to the class of 2011, Ms. Lundin has decided to do a death-defying trick (called The Really Super Amazingly Technical dive). This trick uses a ferris wheel, a moving tub of water, and a stop watch. Starting at the 3 o'clock position, Lundin will be standing on a moving platform on the ferris wheel (which has a radius of 50 ft). The ferris wheel is also moving counter clockwise at 40sec/1rev. Underneath the ferris wheel, there is a moving cart full of water. The cart is 8ft deep and going 15ft/sec on a 240ft long track While the ferris wheel goes around, Lundin will have to jump at the perfect time so she will land in the tub of water and avoid all deathly outcomes. Lundin gave the task of finding the perfect time to jump to her lovely students. Will Lundin survive? Or will her students fail her once again?



(picture not to scale)

The final equation=

$$-240+15\left(\sqrt{\frac{57+50\sin 9w}{16}}+w\right)=50\cos 9w$$

The goal of solving this equation is to get both sides to equal each other. This may seem like a huge scary equation, but it's actually made up of 4 smaller equations.

1. Height of Lundin=57+50sin9w

To understand where Lundin will be on the ferris wheel at any given time, we had to find an equation of the ferris wheel platform (where Lundin is standing)
We determined the circumference of the ferris wheel using the equation **c=2πr**
The wheel has a radius of 50ft, so we filled that into the equation **c=2π(50)** and solved it on a calculator and got **c=314.159ft**

Next, we needed to find how fast (in feet per second) the ferris wheel was going. We already knew that the **one entire rotation for the ferris wheel took 40sec** and we had just figured out the circumference of the wheel (314.159), so we plugged those into an equation that looked like:

$$\frac{314.159\,ft}{40\sec}=\frac{X}{1\sec}$$

And solved by dividing 314.159 by 40. The solved equation looked like:

$$\frac{314.159\,ft}{40\sec}=\frac{7.853\,ft}{1\sec}$$

Finally, we figured out the **Angular Speed** of the ferris wheel. The angular speed is different than feet per second because it tells us what **angle** the platform is at any given moment.
Circles are **360 degrees/1 rotation**. The ferris wheel is moving **40sec/1 rotation**. We plugged this into an equation that looked like:

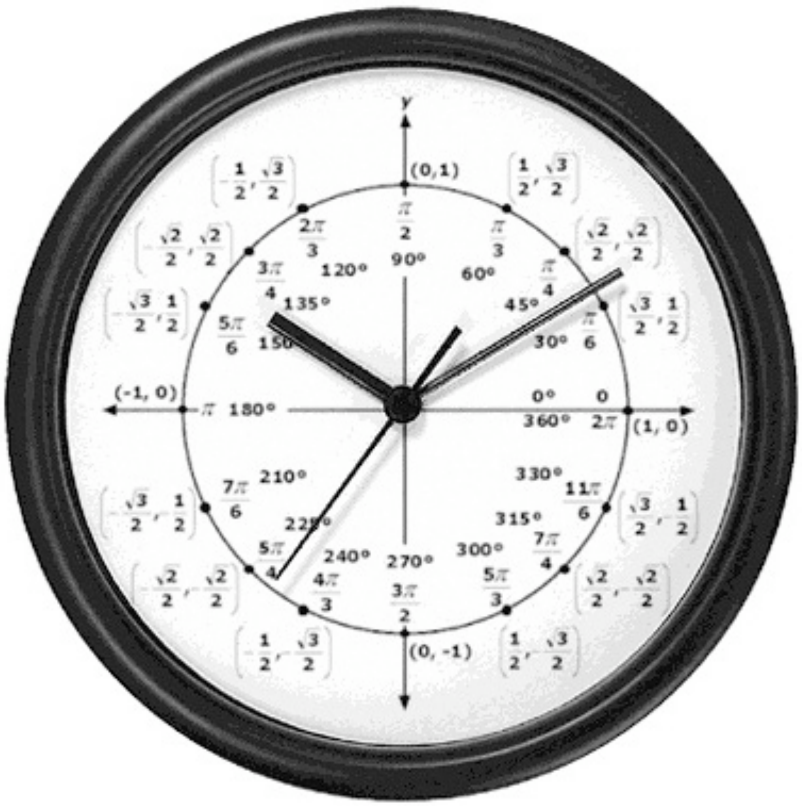
$$\frac{360\deg rees}{40\sec}=\frac{x\deg rees}{1\sec}$$

And solved it just like we did for the Feet Per Second equation (dividing 360 by 40).
The solved equation looked like:

$$\frac{360\deg rees}{40\sec}=\frac{9\deg rees}{1\sec}$$

-NOTE: REMEMBER THE FERRIS WHEEL IS MOVING-
-COUNTER-CLOCKWISE-

In order to test our equations, we solved example problems such as:
How many seconds does it take for the platform go from the 3 o’clock position, to the 11 o’clock position?
To solve this, we must first look at this **UNIT CIRCLE** (which happens to also be **A CLOCK**):



This unit circle shows us the **3 o’clock position=0 degrees** and the **11 o’clock position=120 degrees**

We plugged it into this equation:

$$\frac{x \deg rees}{9 \deg rees} = ?$$

This equation will tell us the **amount of time** it would take for the platform to get to a certain place
X degrees asks for the position we’re looking for (in this case, 120 degrees (aka the 11 o’clock position)
9 degrees is underneath because it’s the degrees per second.

We plugged in 120 and solved:

$$\frac{120\deg rees}{9\deg rees} = 13.333$$

Then we needed to find an equation to tell us the platform height (aka Lundin's height). In this equation, we needed to use SIN

WHAT IS SIN?

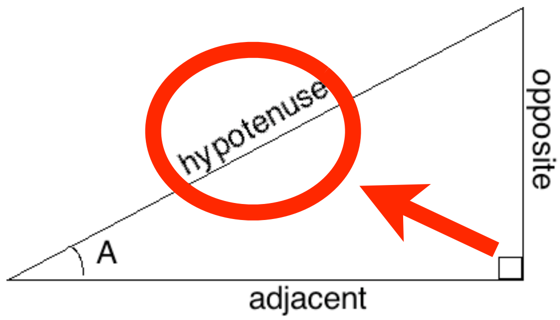
This is a trigonometric function. **SIN** (pronounced sign) is opposite/hypotenuse (explained below).

There are two other trig functions called **COS** (adjacent/hypotenuse) and **TAN** (opposite/adjacent)

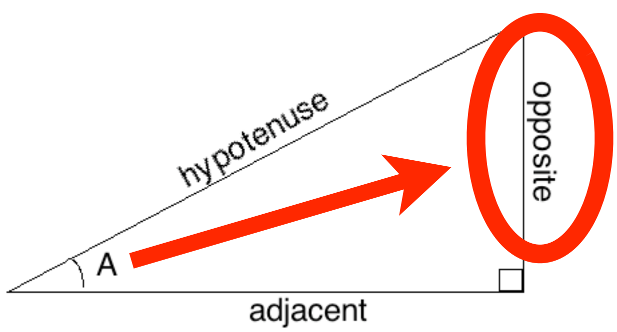
What is Opposite Adjacent and Hypotenuse?

Opposite, Adjacent, and Hypotenuse are the three parts of a right triangle.

The **hypotenuse** is located across from the 90 degree angle of a right triangle.

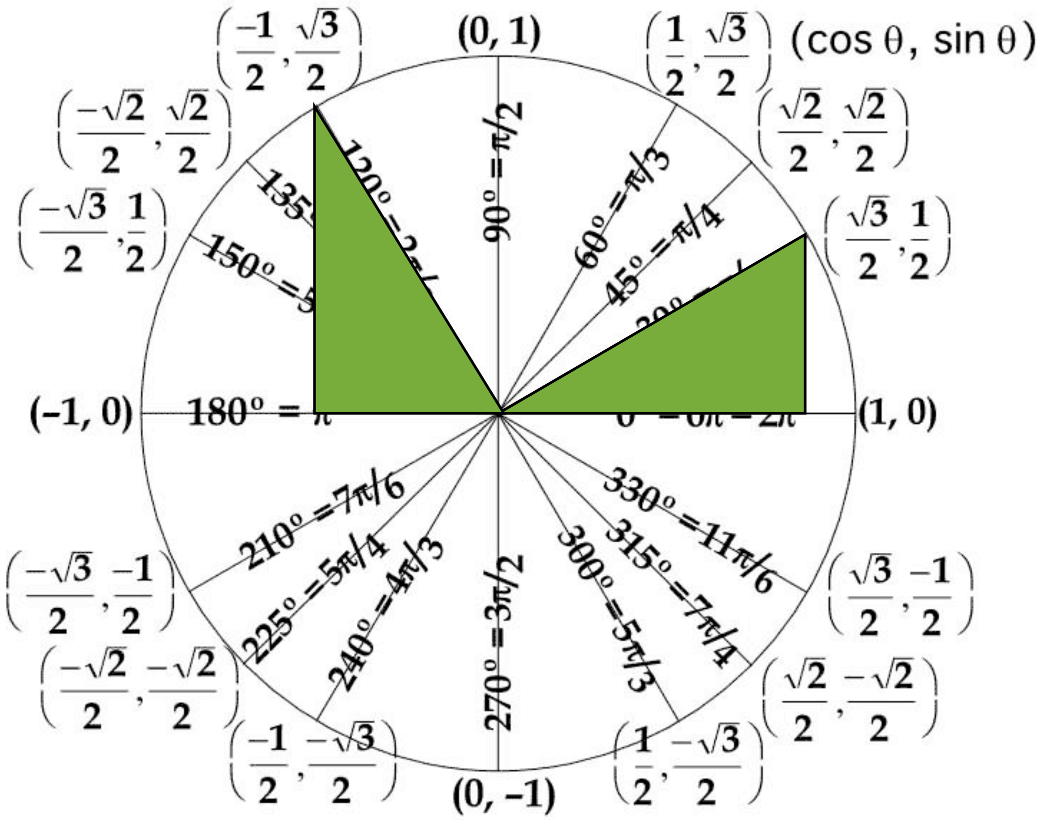


The Opposite is located opposite from the angle.

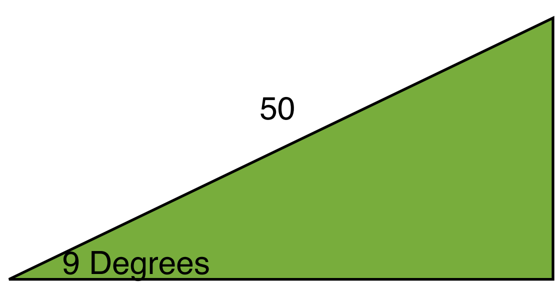


The Adjacent is (simply) the last remaining side of the triangle.

Imagine the unit circle as a bunch of right triangles, if you can't imagine it, here's a picture:



In order to find Lundin's height, we have to use SIN (explained on the last page) and used it to find some example equations such as:
What is the platform's height off the ground 1 second after passing the 3 o'ckick position?
The 3 o'clock position is located at (1,0), or 0 degrees (Lundin's starting point). We learned that the platform is moving 9 degrees/1 second. So if the platform has left the starting point for 1 second, it has gone 9 degrees, which means we will be finding SIN(9).
THE ANGLE OF THE RIGHT TRIANGLE will also be 9, because that's how many degrees the platform has moved
THE HYPOTENUSE OF THE RIGHT TRIANGLE will be 50, because that's the radius of the ferris wheel.



To find Lundin's height we will solve:

$$sin(9) = \frac{X}{50}$$

X=7.821723252011545

But we're not done!
We have to add 65 to the equation because the ferris wheel is 65ft tall from the ground to the middle of the ferris wheel.

7.821723252011545+65=**72.821**

NOW WE MUST GENERALIZE ALL OF THIS WORK INTO ONE EQUATION

The ferris wheel is between 0<W<10 and height is a function of time because height is dependent on the time.

Which brings us the the final equation:
50sin(9w)+65 (W stands for Wheel time aka how long the ferris wheel has been going around)
We had to subtract 8 from 65 because the cart is 8 feet tall which gets us

50sin(9w)+57

2. X Coordinate of Lundin=50cos9w

This equation is very simple to find.
SIN=Y AND COS=X
We already found SIN(the Y coord.) and now we must use COS (because it's used for the X coord.)
We took 50 (the radius of the circle) and multiplied it by COS(9w). 9 is in there because that's how many degrees Lundin is moving per second (just like Lundin's Y coord.) and W is still wheel time (how long the ferris wheel has been going around). Put it all together and it creates

50cos9w

3. The carts horizontal position=-240+15(F+W)

This equation is also pretty simple.
The cart starts at 240ft. to the LEFT of the center of the ferris wheel. If we think of the track as X and the center of the ferris wheel as Y (like a graph), the cart is at -240.
The cart's speed is 15ft/1sec

A linear expression is y=mx+b, we used this as a base for this equation. We used Linear because the speed of the cart is a constant, and doesn't fluctuate like a trigonometric equation would.
The base (b) is -240 (the carts starting time)
Rise/run (m) is 15 (how fast the cart is moving)
All together it creates

y=-240+15x(F+W)

We added F-fall time(explained below) and W-wheel time because

_4._ Fall time= $f=\sqrt{\frac{h}{16}}$

Falling objects have constant acceleration.
The instantaneous speed of a freely falling object increases at approximately 32 ft. per second for each second of its fall.

Time	Speed
0	0
1	32ft/sec
2	64ft/sec
3	96ft/sec
4	128ft/sec
5	160ft/sec

We found the average speed for the first five seconds
0+160/2=**80ft/sec**
And then found how far the object fell in the first five seconds by solving the equation
distance=rateXtime
The rate is the average speed (80)
And the time is how long it's been falling (5)
distance=80x5
And solved
distance=400
This means in five seconds, the object fell 400 feet

Then we found an equation for the average speed.
We substituted t for time and found the average between the instantaneous speed at 0 seconds (t=0, instant speed=0) and the instantaneous speed at the end of t seconds (instantaneous speed=32t).
We found the average speed by solving
0+32/2=16

And found that the average speed over the first t seconds of the fall was 16t.
Since distance=rateXtime we substituted rate with our average speed of 16t which creates the equation
d=16tXtsec
And solved
d=16t^2
Therefore, if an object is free falling, it will fall 16t^2 in the first t seconds.

We then had to add height to the equation. We need to add height because we want to find out how far Lundin is falling from her platform at any given time. This is helpful because she's on a moving platform, and we can figure out if she will fall before or after the cart has passed.
If the object starts from a height of h feet, we need to find what the height is after t seconds.
We once again used the equation d=rXt, but we changed it so d is substituted for h (height).
We plugged in our rate (16t) and our time (t) and solved
h=16t×t
h= $16t^2$
Then we solved it to get t by itself
We divided both sides by 16

$$\frac{h}{16}=t^2$$

And square rooted both sides

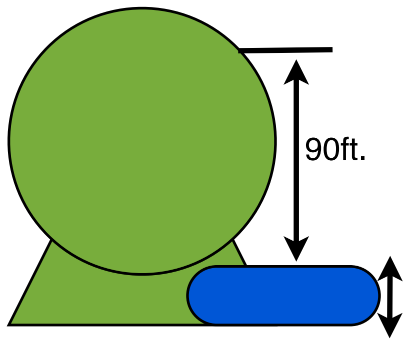
$$\sqrt{\frac{h}{16}}=t$$

Which created our final equation for fall time

$$t=\sqrt{\frac{h}{16}}$$

And then changed t to f (to make it clear that this was for FALL TIME)

$$f=\sqrt{\frac{h}{16}}$$



We tested this final equation on a simplified version of the jump.
Imagine that the ferris wheel had a fixed platform at 90ft above the ground, and the diver free falls into a cart of water that it 8ft above the ground.
(Picture not to scale)

Since the height of the cart is 8ft off the ground, is subtracts from the 90ft of height. Therefore the height of the fall is 82ft.
We substituted 82ft for h in the equation (because h=height and 82ft is the height of the platform) and solved

$$f=\sqrt{\frac{82}{16}}=\sqrt{5.125}=2.263$$

Which means **the diver will reach the water in 2.263 seconds**

Final Equation

Our final equations is composed of all four equations that we have created.

(H)Height of Lundin=57+50sin9w

(XL)X Coord. of Lundin=50cos9w

(XC)X Coord. of Cart=-240+15(f+w)

(F)Fall Time= $\frac{h}{16}$

(Plus X coord. of the cart=x coord. of Lundin)

Since the X coord. of the cart is equal to the x coord. of Lundin we started with those two equations which created
-240+15(f+w)=50cos9w
The equation is equal because in order to solve it, both sides need the equal each other. When you get both sides to equal, the number found is the estimated time Lundin should jump.
Then substituted f with our fall time which created

$$-240+15\left(\sqrt{\frac{h}{16}}+w\right)=50\cos 9w$$

And then substituted h with our height of Lundin which created

$$-240+15(\sqrt{\frac{57+50\sin 9w}{16}}+w=50\cos 9w$$

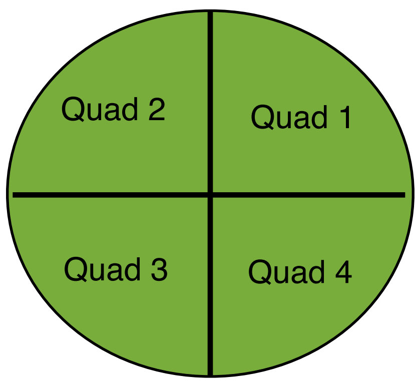
(also known as, OUR FINAL EQUATION)

Solving the equation:

The answer to this equation is **12.283 seconds**
There are three ways to find the answer to this equation, graphically, using a table, and guess and check.
For this I'll only show you two of those three ways.

Using Guess and Check:

Guess and check is the art of plugging in numbers into the equation, solving the equation, and seeing how close you are the answer.
For this answer to work, we need both sides of our equation to be the same.
I started our with using w=10.5 I used 10.5 because I figured it wouldn't happen in the first quadrant (the first 10 seconds) of the unit circle/ferris wheel.



For a guess and check example, we're going to plug in 11 and see what we get. Since w=11 we will replace every w in the equation with 11

$$-240+15(\sqrt{\frac{57+50\sin 9w}{16}}+w=50\cos 9w$$

And after the process of solving the equation step by step we got:

-36.321=-7.821

Since -36.321 does NOT equal -7.821, 11 isn't the answer to this equation.
I kept on guessing and checking by .5 until I got really close (at 12) and then made the number higher little by little until I got to 12.283

Using a Table:

Guess and checking can also be incorporated with tables.

To make everything you've found easier to read (instead of having pages of answers to equations on a billion pages).
To create the guess and check table you keep on guess and checking, and put your answers into a table like this:

W	X Coord. of Cart	X Coord. of Lundin
10.5	-43.755	-3.923
11	-36.321	-7.821
11.5	-28.961	-11.672
12	-21.657	-15.450
12.5	-14.407	-19.134
12.25	-18.025	-17.305
12.265	-17.808	-17.416
12.268	-17.763	-17.438
12.278	-17.625	-17.512
12.28	-17.59	-17.53
12.283	-17.55	-17.55

Another way to do the table is to plug the equations into a graphing calculator and hitting the table button. This will give you a table of every coordinate. You can scroll through the list until you find where the X Coord. of the Cart and the X Coord. of Lundin match (this is also a way to cheat the system and do math really quickly).

Reflection:

Working in a group of two really helped me solve this problem. I worked with Richard Huynh through most of the worksheets and we got most of it done way before the class. Richard really has an understanding of math that if I ever got stuck or confused he would explain what happened and how we got there step by step. I think that if I had went along with the class going through this problem I wouldn't have understood it as well. I think that one on one work when it comes to math just clicks with me. I did struggle with ending up with an equation or solution and not remembering how I got there. I got through this by asking Richard or Lundin for help and writing down every step or the equation that got us there on the side of the paper so I wouldn't forget it next time. This really helped with writing this paper because when I was at home working on it I wasn't confused by what I was reading on my paper.

The three factors that could change the answer to this problem are how fast the cart is moving, how tall the ferris wheel is, and the ferris wheel going clockwise. If the cart was moving faster Lundin would have to jump off the ferris wheel earlier, and if the cart was moving slower Lundin would have to jump off later. If the ferris wheel was taller, Lundin would have a longer fall time, so she would have to jump off earlier. If the ferris wheel was shorter, Lundin would have a shorter fall time and would have to jump off later. If the ferris wheel was to go clockwise (instead of counter-clockwise) I think Lundin would have to jump off around the same area (the 10-20 second range) because she's going the same amount of distance in the same amount of time but would just have a smaller fall time because she's traveling around the bottom of the ferris wheel instead of the top. The real world problems that could happen (and we ignored) are the arch of Lundin's jump and wind speed. We also ignored that if Lundin made it into the cart of water she would hit the side and be really hurt because the cart is in motion and gravity would throw her to the side (I hope this makes sense).

In this problem I used SIN, COS, and TAN which I had learned about in 10th and 11th grade. Although I hadn't mastered SIN, COS, and TAN in the past years this problem helped me get a deeper understanding of them through the worksheets handed out. Before I probably wouldn't be able to tell you how to find COS of anything (not even on a calculator) but now I can give you the definition using triangles AND solve it on the calculator. I also really enjoyed learning about the unit circle, and was a little bit too amazed when I found out about the triangles within the circle.

By using the equations and methods in this problem, I could find out a lot of real world problems. I could find out how long it'll take for someone to fall somewhere using the fall time equation. I could find out where you are at any given point on a ferris wheel (or any spinning wheel). I could tell you how fast your falling if you give me the amount of time you fell. All of these things are very useful and I will use them all the time to impress people with my knowledge.